DI Physics Hons.

Paper-I

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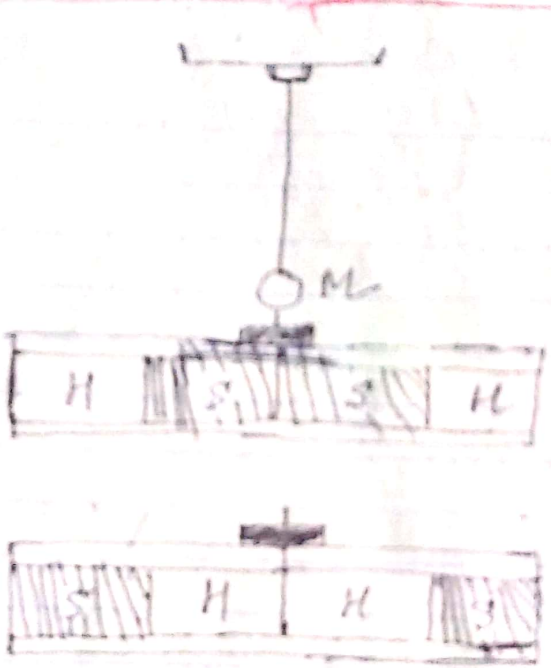

Fig. (1)

The method, using Maxwell's needle, for the determination of modulus of rigidity of the material of a wire is a dynamical method. Maxwell's needle consists of a long hollow brass tube of length l, suspended horizontally by a vertical wire, whose modulus of rigidity is to be determined. The upper end of the wire is clamped to a rigid support.

four brass cylinders, two hollow and two solid, of identical lengths and diameters, can be fitted inside the hollow brass tube. The length of each cylinder is L/4 and when they are placed in end to end, they just fill the hollow tube completely. To facilitate the counting of vibrations of the Maxwell's needle, a small plane mirror is attached to the specimen wire and observations are taken with the help of telescope and scale arrangement.

Theory and Method:

First, the solid cylinders S,S are placed in the inner positions and the hollow cylinders H,H in the outer positions as shown in Fig. ①. The system is allowed to perform torsional oscillations. Let T_1 be the time period of vibration then

$$T_1 = 2\pi\sqrt{I_1/C} \quad \text{--- (i)}$$

where I_1 is the moment of inertia of the suspended system about the wire as axis and C the torsional couple per unit twist of the wire.

Next, the positions of the hollow and solid cylinders are interchanged. Now, if I_2 be the moment of inertia of the system about the wire as axis, then the time period, is given by

$$T_2 = 2\pi\sqrt{I_2/C} \quad \text{---(ii)}$$

From equs. (i) and (ii), we get

$$C = \frac{4\pi^2(I_2 - I_1)}{T_2^2 - T_1^2}$$

But $C = \frac{n\pi r^4}{2l}$,

where l and r are the length and radius of the specimen wire respectively.

$$\therefore \frac{n\pi r^4}{2l} = \frac{4\pi^2(I_2 - I_1)}{T_2^2 - T_1^2}$$

or, $n = \dfrac{2\pi l\,(I_2 - I_1)}{(T_2^2 - T_1^2)\cdot r^4}$ ---- (iii)

Let I_0, I' and I'' be the moments of inertia of the hollow brass case, solid and hollow cylinders respectively about the axes passing through their centres of gravity and perpendicular to their lengths.

As the length of each cylinder is $L/4$, moment of inertia of the needle I_1 about the wire as axis, when the hollow cylinders are placed near the ends of the hollow brass tube, is given by

$$I_1 = I_0 + 2\left[I' + m_1\left(\frac{L}{4\cdot 2}\right)^2\right] + 2\left[I'' + m_2\left(\frac{3L}{4\cdot 2}\right)^2\right]$$

or $$I_1 = I_0 + 2I' + 2I'' + 2m_1\left(\frac{L}{8}\right)^2 + 2m_2\left(\frac{3}{8}L\right)^2 \quad \text{---- (iv)}$$

where m_1 is the mass of each solid cylinder and m_2 that for each hollow ones.

Similarly, moment of inertia of the system I_2, when the solid cylinders are at ends, about the wire as axis is

$$I_2 = I_0 + 2I' + 2I'' + 2m_1\left(\frac{3L}{8}\right)^2 + 2m_2\left(\frac{L}{8}\right)^2 \quad \text{---- (v)}$$

Subtracting eqn. (iv) from (v), we get

$$-I_1 + I_2 = (m_1 - m_2) L_4^2/4 \quad --(vi)$$

Substituting the value of $(I_2 - I_1)$ from eqn. (vi) in eqn. (iii), we have,

$$\eta = \frac{2\pi (m_1 - m_2) L_0^2 \cdot l}{(T_2^2 - T_1^2) r^4} \quad --(vii)$$

The time period T_1 and T_2 in the two cases are calculated with the help of telescope and scale arrangement. The length of the tube L_4 and length of the wire l are measured by a metre scale. Masses of the cylinders, m_1 and m_2, are obtained by weighing. The radius of the wire r is measured with the help of a screw gauge at several places on the wire in two perpendicular directions and its mean value is calculated. Thus, substituting the values of various quantities, in (vii), we can calculate the modulus of rigidity of the material of wire. Maxwell's needle

has the following two advantages over torsional pendulum:

(i) In Maxwell's needle, the load suspended at the end of the wire remains the same since only the positions of solid and hollow cylinders are interchanged. Thus, the couple per unit twist due to torsional reaction remains constant.

(ii) Here, the calculation of $(I_1 - I_2)$ reduces to the determination of the difference between the masses of solid and hollow cylinders and the length of the hollow tube can be measured accurately and easily.